

Study of the Influence of the Number Normalization Scheme Used in Two Chaotic Pseudo Random Number Generators Used as the Source of Randomness in Differential Evolution

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Abstract. In many publications, authors showed that chaotic pseudo random number generators (PRNGs) may improve performance of the evolutionary algorithms. In this paper, we use two chaotic maps Gingerbread man and Tinkerbell as the chaotic PRNGs instead of the classical PRNG in the differential evolution. Numbers generated by this maps are normalized to the unit interval by three different methods – operation modulo, straightforward number normalization where we know minimal and maximal generated number and arctangent of the two variables x and y , where numbers x and y are generated by the Gingerbread man map and Tinkerbell map. The first goal of this paper is to show whether the differential evolution convergence speed might be affected by the way how we normalize number generated by the chaotic map. The second goal is to find out the influence of the probability distribution function of the selected chaotic PRNGs. The results mentioned below showed that the selected normalization method may improve differential evolution convergence speed, especially in the case of arctangent and straightforward number normalization, where we know the minimal and maximal generated numbers.

Keywords: Differential evolution, pseudo random number generator, number normalization scheme, chaos, Gingerbread man, Tinkerbell

* The following grants are acknowledged for the financial support provided for this research: Grant Agency of the Czech Republic - GACR P103/13/08195S, is partially supported by Grant of SGS No. SP2014/42, VB - Technical University of Ostrava, Czech Republic, by the Development of human resources in research and development of latest soft computing methods and their application in practice project, reg. no. CZ.1.07/2.3.00/20.0072 funded by Operational Programme Education for Competitiveness.

1 Introduction

Interconnection between chaos and randomness is known very long time. In 1940, J. von Neumann used logistic map as the chaotic pseudo random number generator (cPRNG). From this year, chaos as the PRNG has been used in various research areas: cryptography ([23], [9]); image encryption ([11], [4]); new PRNG research ([14], [26]); evolutionary algorithms (EAs). In the last mentioned area, chaos have been used successfully as the chaotic PRNG (cPRNG) for example in the bee colony algorithm ([1]), particle swarm optimization (PSO) ([17]), genetic algorithm ([27]) or in differential evolution (DE) ([10]). R. Caponetto et al. used logistic map as the cPRNG in all phases of EA, where the random number is needed ([3]). G. Zilong et al. described a novel immune EA, where logistic map is used to generate the chaos sequence ([28]). B. Liu et al. used logistic map to improve PSO ([15]). The utilization of analytic programming for a synthesis of control law for selected discrete chaotic systems is described where authors used logistic map and then Henon map ([20]). B. Alatas introduced twelve chaos-embedded PSO methods, where eight chaotic maps have been analyzed in the benchmark functions ([2]). The same author used logistic map in the chaotic harmony search algorithm. Senkerik et al. used DE for the evolutionary tuning of controller parameters for the stabilization of different chaotic systems, where the selected controlled discrete chaotic systems (Burgers map, Delayed logistic map and Lozi map) are used also as cPRNGs to drive the mutation and crossover process ([21]). L. dos Santos Coelho and V. C. Mariani described PID controller tuned by firefly algorithm using Tinkerbell map ([8]).

The motivation of this paper is that in the most publications dealing with the chaos powered EA there is not made clear whether the improvement of the EAs convergence speed stems from the uniqueness of the sequence of the numbers generated by the cPRNGs or by the probability distribution function (PDF) of the selected cPRNGs. In the most publications, authors use only one way of the number normalization and we spared the comparison of the number normalization schemes.

In our work, we used two-dimensional maps – Gingerbread man and Tinkerbell as the cPRNGs in DE. Numbers generated by cPRNG are used in all cases where the randomness is needed in DE. As Gingerbread man and Tinkerbell maps might generate numbers outside of the unit interval, we have selected three schemes of number normalization. Operator modulo, normalization by bounds are traditional representatives of common normalization schemes. We have defined the third scheme in addition to overcome some peculiarities related with the preceding two schemes (eg. we need to know the bounds, modulo leads to over utilization of some sub-intervals of the generator). Normalized number is then used in DE and the convergence speed of DE to the global minimum is observed.

The rest of the paper is organized as follows: The differential evolution is described briefly in the Section 2. In the Section 3, the selected cPRNGs are introduced. The selected testing problems are mentioned in the Section 5. In the Section 4, we are presenting our methods of number normalization. The

methods of the analysis and setting of the DE are mentioned in the Section 6. In the Section 7, the results of the experiments are recorded and in the Section 8, the experiments results are discussed.

2 Differential evolution

DE belongs to the family of the evolutionary algorithms (EAs) working with the population of the individuals ([18, 24, 25]).

Here, let us describe the DE informally. The first population is generated randomly in the space of possible solutions. Then for each individual three random individuals (parents) are chosen. From these parents, we create a noise vector \mathbf{v} according to the following equation

$$v_j = x_{r_3,j}^G + F(x_{r_1,j}^G - x_{r_2,j}^G), \quad (1)$$

where v_j denotes j -th parameter of the noise vector, $x_{r_3}^G$ is the third randomly selected parent, $x_{r_1}^G$ is the first randomly selected parent and $x_{r_2}^G$ is the second randomly selected parent. The superscript G means the actual generation, F denotes mutation constant.

Then random number r from the unit interval is generated for each parameter of the actual individual. If $r < CR$, where CR is crossover probability, parameter from the noise vector is added to the trial individual, otherwise parameter from the actual individual is chosen. Now, the fitness value of the trial individual is computed. If it is better than fitness of the actual individual, the trial individual will be added to the new population, otherwise actual individual will be added. Process described above is repeated until some criterion of convergence is reached ([25]).

Beside the first variant of DE characterized by the Eq. (1) we have also included the variant DE/best/1/bin. The noise vector creation is described by the following equation

$$v_j = x_{\text{best},j}^G + F(x_{r_2,j}^G - x_{r_3,j}^G), \quad (2)$$

where x_{best}^G denotes the individual with the best fitness value in the actual generation ([19]).

3 Chaotic maps

This section contains description of the selected chaotic maps used as the cPRNGs in DE. We have selected Gingerbread man and Tinkerbell which might generate numbers outside the unit interval. In addition, they give promising results from the view of DE convergence speed and they are easy to implement.

3.1 Gingerbread man map

The Gingerbread man map (see Fig. 1) is a chaotic two-dimensional map which was studied by R. Devaney ([6]) since 1984 and is given by the following equation

$$\begin{aligned}x_{n+1} &= 1 - y_n + |x_n|, \\ y_{n+1} &= x_n.\end{aligned}\tag{3}$$

In this paper, the initial values of x and y have been experimentally set to $x_0 = 9.0$ and $y_0 = 3.7$.

3.2 Tinkerbell map

Tinkerbell map (see the Fig. 2) is the strange attractor with a fractal basin boundary and it was proposed by H. E. Nusse and J. A. Yorke ([16]). It is given by the following equation

$$\begin{aligned}x_{n+1} &= x_n^2 - y_n^2 + ax_n + by_n, \\ y_{n+1} &= 2x_ny_n + cx_n + dy_n.\end{aligned}\tag{4}$$

In this paper the parameters have been set to $a = 0.9$, $b = -0.6013$, $c = 2.0$, and $d = 0.5$ as suggested in ([16]). The initial value of x and y have been experimentally set to $x_0 = 0.1$ and $y_0 = -0.1$.

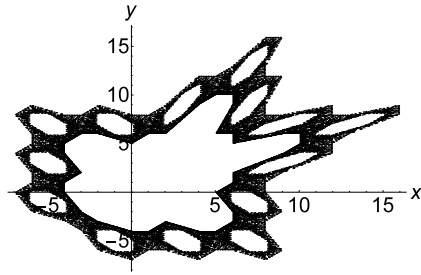


Fig. 1: Gingerbread man map

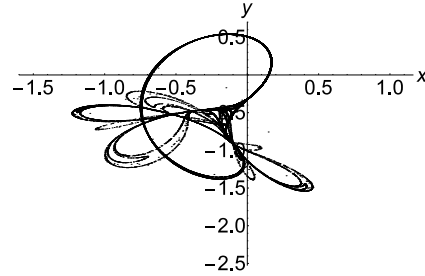


Fig. 2: Tinkerbell map

4 Normalization of the number generated by the cPRNG

We have selected two discrete dynamical systems – Gingerbread man and Tinkerbell to be used as the cPRNGs in DE. Because these cPRNGs might generate numbers outside the unit interval it is necessary to normalize that number to the unit interval. In our work, we have chosen operation modulo (Modulo), straightforward number normalization where we know minimal and maximal generated number (Bounds) and two-argument variant of arctangent (Atan2) where the

real numbers x and y are generated by the Gingerbread man and Tinkerbell maps. Subsequently, we have modified the uniform PDF of Mersenne Twister (MT) to approximate the PDF of our cPRNGs. In the following paragraphs, we would like to clarify the reasons why we have chosen these three ways of number normalization.

As the first way, operation *modulo* has been chosen. It is the easiest way how to normalize the numbers lying outside of the unit interval. Each number generated by the cPRNG is modified according to the following equation

$$z_i = |n_i| \mod 1, \quad (5)$$

where n_i is number generated by the selected cPRNG, *mod* denotes operator modulo and z_i is the i -th normalized number. This way of number normalization has been successfully used for example by D. D. Davendra et al. ([5]), where authors use Tinkerbell and others as the cPRNGs in a scatter search algorithm. The main problem of this scheme is its PDF, because different numbers generated by cPRNGs might be normalized to the same values. For example the sequence $\{1.2, 2.2, 3.2, \dots\}$ will be normalized to the single value 0.2.

The second way how to normalize the number generated by the cPRNG to the unit interval is the straightforward number normalization where we know minimum and maximum generated by the number generator (Bounds). This normalization scheme is given by the following equation

$$z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)}, \quad (6)$$

where $x = (x_1, \dots, x_n)$ and z_i is the i -th normalized number. This scheme of normalization has been successfully used by L. dos Santos et al. ([7,8]). The main problem of this way was that we do not know the minimal and maximal number generated by the Gingerbread man and Tinkerbell map. Due this bottleneck, for each chaotic map we had generated one billion numbers with accuracy to one hundred decimal places and minimal and maximal values were obtained.

The last scheme is the arctangent $\text{atan2}(y, x)$, which has been experimentally added. The main advantage is that there is not distortion of the PDF like in the case of *modulo*. Function $\text{atan2}(y, x)$ computes the angle in the sampling plane corresponding to the phase angle of the point (x, y) . The number generated by the chaotic map is then modified according to the following equation

$$z_i = \text{atan2}(y_i - \bar{y}, x_i - \bar{x}), \quad (7)$$

where z_i denotes the i -th normalized number, y_i is the y coordinate of the selected chaotic map, x_i is the x coordinate of the selected chaotic map and (\bar{x}, \bar{y}) denotes the coordinates of the attractor center equaling to the center computed as the average of the samples generated in the preceding step.

5 Selected testing problems

We have selected nine functions from the CEC2013 benchmark ([13]). For the specification of the selected function, please see ([13]). The reason of this choice is that CEC2013 benchmark provides 28 difficult functions and in the future we would like to extend our work to other functions from this benchmark and from the CEC2014 benchmark, see ([12]).

Table 1: Selected functions from benchmark CEC2013

Cathegory	Funct. Name		Global min.
Unimodal	f_1	Sphere	-1400
	f_5	Different Powers	-1000
Basic multimodal	f_9	Rotated Weierstrass	-600
	f_{13}	Non-Continuous Rotated Rastrigins	-200
	f_{15}	Rotated Schwefel's	100
	f_{16}	Rotated Katsuura	200
	f_{17}	Lunacek Bi Rastrigin	300
	f_{22}	Composition function 2	800
Composition	f_{23}	Composition function 3	900

6 Methods and experiment settings

In the first experiment, we have used Gingerbread man and Tinkerbell maps as the cPRNGs with the different number normalization schemes in DE/best/1/bin and DE/rand/1/bin. As the testing problems nine functions from CEC2013 have been chosen. For each function, three categories have been created according to the number normalization scheme described above and denoted as **Atan2**, **Bounds** and **Modulo**. Each experiment has been repeated fifty times. The results are reported in Tables 3 – 14. The results represents the relative number of winnings of the given normalization scheme used in the cPRNG from the view of DE convergence speed. When two number normalization schemes reach the best results in the same time, they are recorded both as the best.

Now we would like to make clear motivation of the following experiments. The first goal was to find out which normalization scheme is the most successful from the view of DE convergence speed and the second goal was to investigate the influence of the PDF of the selected chaotic PRNGs. The results of the experiments mentioned in the Tables 3 – 14 (odd columns) give us the relative number of winnings of the normalization scheme using in the selected cPRNG. In our opinion, the greatest mean of the results of the cPRNG corresponds to the most successful normalization scheme and the PDF of the cPRNG fundamentally influences DE convergence speed. We have selected the following statistical

methods to find out the greatest mean of the normalization schemes (columns of the mentioned Tables):

1. It was necessary to find out if the results mentioned in the Tables 3 – 14 are normally distributed. We have used Kologorov-Smirnov test for each column of these results.
2. When the normality is verified we have to find out if the variances of the columns of the mentioned tables can be considered as equal.
3. If the variances can be considered as equal, statistical test ANOVA will be used to find out if the means of columns can be considered as equal.
4. If the variances can not be considered as equal, we can use one-sided and two-sided T-tests with nonequivalent variances to find out which mean is the greatest.
5. When the means can not be considered as equal the statistical one-sided T-tests with equivalent variances can be applied to find out which mean is the greatest.

When the greatest means are found we can compare the means of the columns of the cPRNGs with the columns of MT with the modified PDF according to these cPRNGs. If the normalization scheme fundamentally affects DE convergence speed, the results of the cPRNG and MT with the modified PDF according to this cPRNG will be comparable.

We have assumed that the results of the different number normalization schemes will be different. From this reason it was necessary to formulate the null H_0 and alternative hypothesis H_A and the level of significance α :

- H_0 : The means of results of the categories denoted as Atan2, Bounds and Modulo are **different**.
- H_A : The means of results of the categories denoted as Atan2, Bounds and Modulo are **the same**.
- The significance level has been chosen to be $\alpha = 0.1$ (10%).

Firstly it was necessary to verify whether the results mentioned in Tables 3 – 14 (odd columns) are normally distributed. The Kolmogorov-Smirnov test has been applied and normal distribution has been confirmed. Then we have performed the tests of variance equality, where Atan2, Bounds and Modulo have been compared with each other. It was found that we can consider the variances of the categories as equal in the three of four cases. In the case of DE/rand/1/bin powered by cPRNG using Gingerbread man the variances of Atan2, Bounds and Modulo can not be considered as equal. To compare means of the remaining data sets statistical method ANOVA has been applied. The results are mentioned in the Table 15. We can see that in the case of DE/best/1/bin powered by cPRNG using Gingerbread man map means can be considered as equal. That means the success of all three normalization schemes is comparable.

Statistical one-sided (if it was necessary two-sided) T-test has been performed for data sets DE/best/1/bin powered by the cPRNG using Tinkerbell, and DE/rand/1/bin powered by the cPRNG using both chaotic maps.

For each case the null hypothesis has been formulated according to the results mentioned in the Tables 3, 4, 7, 8, 11 and 12 for DE/best/1/bin and 5, 9, 13 for DE/rand/1/bin. The significance level has been chosen to be $\alpha = 0.1$ (To save space DE/best/1/bin will be denoted as DE/best and DE/rand/1/bin as DE/rand):

- DE/best, Tink.: H_0 : The mean μ_A of Atan2 is greater than the mean μ_B of Bounds and mean μ_M of Modulo.
- DE/best, Ging.: H_0 : The mean μ_B of Bounds is greater than the mean μ_A of Atan2 and mean μ_M of Modulo.
- DE/rand, Tink.: H_0 : The mean μ_B of Bounds is greater than the mean μ_A of Atan2 and mean μ_M of Modulo.

The results of the T-tests for cPRNGs are mentioned in the Table 16. In the case where we have denied the null hypothesis that one tested normalization schemes has greater mean than the second one we have used the two-sided T-test to find out if the means can be considered as equal and in the column p-value we mention the value from the two-sided T-test (denoted by (t_2)), else p-value from the one-sided T-test is mentioned. The last column denotes if the null hypothesis H_0 has been accepted (Acc.) or denied (Den.).

The goal of the second part of this paper was to find out if the fruitfulness of the cPRNG used in DE is affected just by its PDF or it also depends on the specific sequences of numbers generated by cPRNGs. From this reason we have modified the PDF of MT to generate numbers with the same distribution like our cPRNGs using given chaotic map and particular number normalization Atan2, Bounds, and Modulo. Results are mentioned in the Tables 3 – 14 (even columns). We have verified by Kolmogorov-Smirnov test that the results mentioned in the columns of the Tables 3 – 14 are normally distributed. ANOVA could not be used in DE/best/1/bin powered by MT with the modified PDF according to the cPRNG using Tinkerbelle (tMT), DE/rand/1/bin powered by tMT and DE/rand/1/bin with the modified PDF according to the cPRNG using Gingerbread man (gMT) because variances of data sets could not be considered as equal. In the case of DE/best/1/bin powered by gMT the variances could be considered as equal and the test ANOVA has been applied, see Table 15. Subsequently, we have formulated null and alternative hypothesis and T-test (one-sided as well as two-sided denoted as t_2) has been applied to the remaining data sets:

- DE/best, tMT: H_0 : The mean μ_A^{tMT} is greater than the mean μ_B^{tMT} and mean μ_M^{tMT} . H_A : The mean μ_A^{tMT} is not greater than the mean μ_B^{tMT} and mean μ_M^{tMT} . **Interpretation:** MT with modified PDF according to the cPRNG using Tinkerbelle and normalization scheme Atan2 is the most successful from the view of DE/best convergence speed (in comparison with tMT using Bounds and Modulo).
- DE/best, gMT.: In this case it was found out by the test ANOVA that the means can be considered as equal, see Table 15.

- DE/rand, tMT: H_0 : The mean μ_M^{tMT} is greater than the mean μ_A^{tMT} and the mean μ_B^{tMT} . H_A : The mean μ_M^{tMT} is not greater than the mean μ_A^{tMT} and the mean μ_B^{tMT} . **Interpretation:** MT with modified PDF according to the cPRNG using Tinkerbell and normalization scheme Modulo is the most successful from the view of DE/rand convergence speed (in comparison with tMT using Atan2 and Bounds).
- DE/rand, gMT: H_0 : The mean μ_B^{gMT} is greater than the mean μ_A^{gMT} and the mean μ_M^{gMT} . H_A : The mean μ_B^{gMT} is not greater than the mean μ_A^{gMT} and the mean μ_M^{gMT} . **Interpretation:** MT with modified PDF according to the cPRNG using Tinkerbell and normalization scheme Bounds is the most successful from the view of DE/rand convergence speed (in comparison with tMT using Atan2 and Modulo).

The results are mentioned in the Table 17. In the last column we explicitly indicate whether the null hypothesis H_0 has been accepted (Acc.) or denied (Den.).

To find out whether the PDF of the cPRNG fundamentally affect DE convergence speed we have compared the results of cPRNGs (using number normalization Atan2, Bounds, Modulo) and MT with modified PDF mentioned in the Tables 3 – 14. We have compared the means of the best results if the cPRNG and modified MT reach the best results in the same category (Atan2, Bounds, Modulo). If all categories reach the comparable results we compare means of all categories (μ_A vs. μ_A^{MT} , μ_B vs. μ_B^{MT} ...). If the number generators reach the best results in the different category, we compare these categories at the end of our work (in the case of DE/rand, Ging. vs gMT). It was found out by the F-test that the variances of the comparing results can be considered as equal. We have formulated hypothesis and T-test has been used. The significance level has been chosen to be $\alpha = 0.1$. The results are mentioned in the Table 18.

- DE/best, Tink vs. tMT, Atan2: H_0 : The mean μ_A is equal to the mean μ_A^{tMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/best, Ging. vs. gMT, Atan2: H_0 : The mean μ_A is equal to the mean μ_A^{gMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/best, Ging vs. gMT, Bounds: H_0 : The mean μ_B is equal to the mean μ_B^{gMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/best, Ging vs. gMT, Bounds: H_0 : The mean μ_M is equal to the mean μ_M^{gMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand, Tink. vs. gMT, Atan2: H_0 : The mean μ_A is equal to the mean μ_A^{tMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand, Tink vs. gMT, Bounds: H_0 : The mean μ_B is greater than the mean μ_B^{tMT} . **Interpretation:** We can not say that the sequence of the numbers

generated by the cPRNG does not significantly influence DE convergence speed.

- DE/rand, Tink vs. gMT, Bounds: H_0 : The mean μ_M is smaller than the mean μ_M^{tMT} . **Interpretation:** We can not say that the sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand, Tink. vs gMT, Bounds vs. Modulo: The mean μ_B is equal to the mean μ_M^{tMT} . **Interpretation:** We can not say that the sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand, Ging. vs. gMT: H_0 : The mean μ_A is equal to the mean μ_A^{gMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand, Ging. vs. gMT: H_0 : The mean μ_B is equal to the mean μ_B^{gMT} . **Interpretation:** The sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.

Setting of DE (D denotes dimension, NP number of individuals, F mutation constant, and CR crossover probability):

- $D = 10$: $NP = 50$, $G = 200$
- $D = 20$: $NP = 100$, $G = 400$
- $D = 30$: $NP = 150$, $G = 600$
- For all experiments $F = 0.5$, $CR = 0.85$

7 Results

The results for DE/best/1/bin and DE/rand/1/bin with dimensions $D = 10$, $D = 20$, and $D = 30$ are mentioned in the Tables 3 – 14. The results describe the percentage of experiments, where the selected number normalization scheme applied to the number generated by the cPRNG has been the most successful. In other words in the tables mentioned above we can find the relative number of winnings of the normalization scheme using in the selected cPRNG (from the view of DE convergence speed). For the statistical analysis, please see [22]. The best results for DE using cPRNGs are marked in bold and the best results for DE using MT with the modified PDF are mentioned in the brackets. MT with the modified PDF according to the cPRNG using Gingerbread man map is denoted as gMT and Tinkerbelle map as tMT.

8 Conclusion

In this paper, we have been dealing with the effect of the normalization of the number generated by the chaotic maps to the DE convergence speed. Two chaotic maps – Gingerbread man and Tinkerbell and two types of DE – DE/best/1/bin and DE/rand/1/bin have been used. As the number normalization methods operation modulo (Modulo), straightforward number normalization where we know minimal and maximal generated number (Bounds) and arctangent (Atan2) of the two variables x and y , where numbers x and y are outputs of the Gingerbread man map and Tinkerbell map have been chosen. Two first number normalization schemes have been successfully used in many publications mentioned above. The third scheme has been added to our work because there is no distortion of the PDF like in the case of operation modulo and we did not find the publication, where this scheme is used in this context. The goal of the first experiment was to find out if the normalization scheme used in the cPRNG which may generate numbers outside the unit interval can affect the DE convergence speed and which scheme is the most successful. In the second experiment, we have investigated the effect of the PDF of the cPRNG to the DE convergence speed. The main question was if the convergence speed of DE is influenced just by the PDF of the selected cPRNG or the numbers sequence plays a significant role in this process. From this reason we have modified MT to generate numbers with the same PDF like our cPRNGs using three schemes of the number normalization described above.

In the first experiments we have applied three schemes of number normalization to the numbers generated by Gingerbread man and Tinkerbell map. We have recorded how fast DE using these cPRNGs reaches the best results. If two schemes reach the best results in the same time, they are recorded as the best both. The results are mentioned in the Tables 3 – 14. In the second experiment we have modified MT to generate numbers with the same probability like our cPRNGs. The results are mentioned in the same tables like in the case of the first experiment.

As the most successful normalization scheme the scheme with the greatest mean is considered. Based on the results mentioned in the Tables 3 – 14 we have verified the normal distribution by Kolmogorov-Smirnov test. Then for each data sets – DE/best/1/bin using Tinkerbell, DE/best/1/bin using Gingerbread man etc. we have tested if the variances can be considered as equal. In three data sets where MT with modified PDF had been used (DE/best/1/bin tMT, DE/rand/1/bin tMT and DE/rand/1/bin gMT) the variances could not be considered as equal. In the rest of data sets the statistical test ANOVA has been applied to find out if the means of normalization schemes can be considered as equal. The results mentioned in the Table 15 showed the means of the categories (Atan2, Bounds and Modulo) of the DE/best/1/bin powered by Gingerbread man map and the means of the three categories (Atan2, Bounds and Modulo) of the DE/best/1/bin powered by gMT can be considered as equal.

For data sets where we have found out that the means of categories can not be considered as equal the statistical T-test has been used to find out which

normalization scheme has been the most successful. The results are mentioned in the Tables 16 and 17. In the case of cPRNGs for DE/best/1/bin using Tinkerbelle the normalization scheme **Atan2** has been the most successful. In the case of DE/rand/1/bin using Tinkerbelle and DE/rand/1/bin using Gingerbread man the normalization scheme **Bounds** has been the most successful. In the case of DE/best/1/bin powered by tMT the normalization scheme **Atan2** has been the most successful. In the case of DE/rand/1/bin powered by tMT the normalization scheme **Modulo** has been the most successful and in the case of DE/rand/1/bin powered by gMT with the normalization schemes **Atan2** and **Bounds** has been the most successful.

The last step of our work was to compare the results of the DE powered by cPRNGs using different number normalization and DE powered by MT with modified PDF according to these cPRNGs. When we look at the Table 18 we can make some conclusions:

- DE/best/1/bin, Tink. vs. tMT, Atan2: The means can be considered as equal. There is not significant difference between the cPRNG using Tinkerbelle and number normalization Atan2 and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
- DE/best/1/bin, Ging. vs. gMT, Atan2: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Atan2 and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
- DE/best/1/bin, Ging. vs. gMT, Bounds: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Bounds and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
- DE/best/1/bin, Ging. vs. gMT, Modulo: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Modulo and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
- DE/rand/1/bin, Tink. vs. tMT, Atan2: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Atan2 and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
- DE/rand/1/bin, Tink. vs. tMT, Bounds: The means can not be considered as equal. The cPRNG using Tinkerbelle and number normalization Bounds

- has reached better results (its mean is greater) than MT with the modified PDF according to this cPRNG. **Interpretation:** We can not say that the sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
- DE/rand/1/bin, Tink. vs. tMT, Modulo: The means can not be considered as equal. MT with the modified PDF according to the cPRNG using Tinkerbell and Modulo has reached better results (the mean is greater) than this cPRNG. **Interpretation:** We can not say that the sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
 - DE/rand/1/bin, Tink. vs. tMT, Bounds vs. Modulo: In this case we have decided to compare two best number generators. We have compared cPRNG using Tinkerbell and number normalization Bounds and MT with the modified PDF according to the cPRNG using Tinkerbell and Modulo. The means are comparable. There is not significant difference between the cPRNG using Tinkerbell and number normalization Bounds and MT with the modified PDF according to the cPRNG using Tinkerbell and Modulo. **Interpretation:** We can not say that the sequence of the numbers generated by the cPRNG does not significantly influence DE convergence speed.
 - DE/rand/1/bin, Ging. vs. gMT, Atan2: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Atan2 and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.
 - DE/rand/1/bin, Ging. vs. gMT, Bounds: The means can be considered as equal. There is not significant difference between the cPRNG using Gingerbread man and number normalization Bounds and PRNG MT with the modified PDF according to this cPRNG. **Interpretation:** The sequence of the numbers generated by this cPRNG does not significantly influence DE convergence speed.

From the results mentioned in the Section 7 we can say that the number normalization scheme might influence DE convergence speed. In our experiments, number normalization denoted as Atan2 and Bounds reached the best results. In the second part of our work, we were interested in the influence of the PDF and number sequences of the cPRNG using in DE. From the results mentioned above we can see that in three cases from four we can consider the means of the best results of the cPRNG and MT with the modified PDF according to this cPRNG as the same. That means that there is not significant difference between results of the cPRNG using the certain scheme of number normalization and MT with the modified PDF according to this cPRNG.

In the case of DE/rand/1/bin cPRNG using Tinkerbell and Bounds reached the best results and MT with the modified PDF according to the cPRNG using Tinkerbell reached the best results with number normalization Modulo. On the other hand the results of the cPRNG using Tinkerbell and Bounds reached the

comparable results like MT with modified PDF according to the cPRNG using Tinkerbell and Modulo.

On the base of the results mentioned in the section 7 we express our opinion that the main role of the success of the cPRNG using in DE plays its PDF and the sequence of the numbers generated by this cPRNG is of secondary importance.

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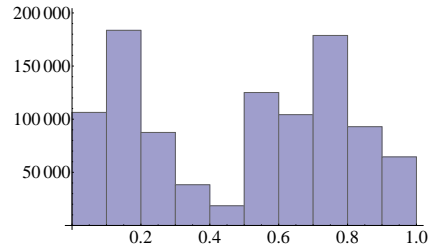


Fig. 3: Histogram of the $1e+6$ samples generated by the cPRNG using Tinkerbell and Atan2 normalizer

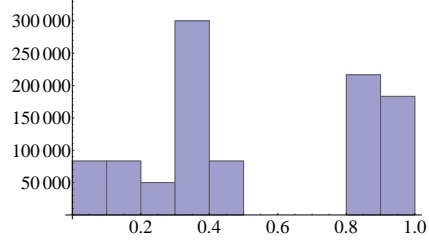


Fig. 4: Histogram of the $1e+6$ samples generated by the cPRNG using Gingerbread man and Atan2 normalizer

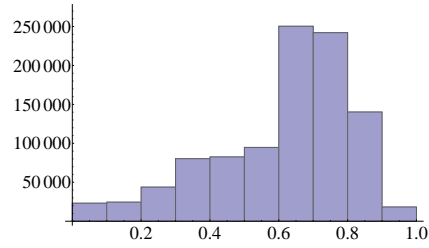


Fig. 5: Histogram of the $1e+6$ samples generated by the cPRNG using Tinkerbell and Bounds normalizer

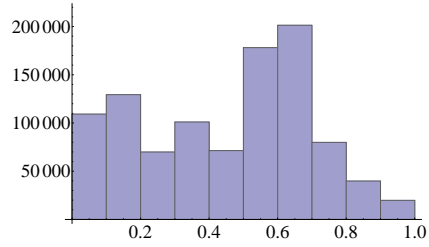


Fig. 6: Histogram of the $1e+6$ samples generated by of the cPRNG using Gingerbread man and Bounds normalizer

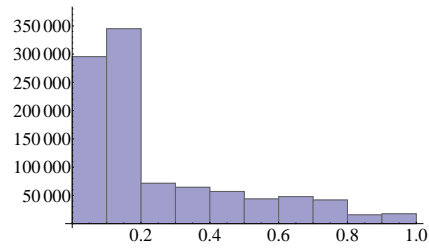


Fig. 7: Histogram of the $1e+6$ samples generated by the cPRNG using Tinkerbell and Modulo normalizer

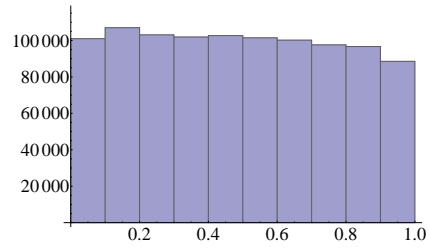


Fig. 8: Histogram of the $1e+6$ samples generated by the cPRNG using Gingerbread man and Modulo normalizer

Table 2: Table of the selected symbols using in the text

Symbol	Meaning
tMT	MT with the modified PDF according to the cPRNG using Tinkerbelle.
tMT, Atan2	MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Atan2.
tMT, Bounds	MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Bounds.
tMT, Modulo	MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Modulo.
gMT	MT with the modified PDF according to the cPRNG using Gingerbread man.
gMT, Atan2	MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Atan2.
gMT, Bounds	MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Bounds.
gMT, Modulo	MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Modulo.
μ_A	The mean of results of the DE using cPRNG using number normalization Atan2.
μ_B	The mean of results of the DE using cPRNG using number normalization Bounds.
μ_M	The mean of results of the DE using cPRNG using number normalization Modulo.
μ_A^{tMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Atan2.
μ_B^{tMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Bounds.
μ_M^{tMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Tinkerbelle and number normalization Modulo.
μ_A^{gMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Atan2.
μ_B^{gMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Bounds.
μ_M^{gMT}	The mean of results of the DE using MT with the modified PDF according to the cPRNG using Gingerbread man and number normalization Modulo.

Table 3: DE/best/1/bin, $D = 10$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	74%	(66%)	20%	34%	6%	0%
f_5	90%	(88%)	4%	12%	6%	0%
f_9	0%	0%	82%	8%	22%	(94%)
f_{13}	22%	28%	36%	38%	44%	(40%)
f_{15}	30%	12%	40%	(44%)	30%	(44%)
f_{16}	2%	0%	68%	12%	40%	(88%)
f_{17}	70%	(46%)	14%	(46%)	16%	8%
f_{22}	84%	(66%)	14%	34%	2%	0%
f_{23}	72%	(58%)	16%	42%	12%	0%

Table 4: DE/best/1/bin, $D = 10$, Gingerbread man (G) vs. gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	68%	(50%)	0%	0%	32%	(50%)
f_5	92%	(100%)	0%	0%	8%	0%
f_9	0%	0%	100%	(100%)	0%	0%
f_{13}	10%	14%	64%	(50%)	28%	42%
f_{15}	6%	6%	70%	(76%)	24%	18%
f_{16}	0%	0%	96%	(94%)	4%	6%
f_{17}	58%	(64%)	4%	14%	38%	22%
f_{22}	96%	(60%)	0%	6%	4%	34%
f_{23}	92%	(68%)	0%	4%	8%	28%

Table 5: DE/rand/1/bin, $D = 10$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	32%	32%	44%	(64%)	24%	14%
f_5	16%	20%	44%	(54%)	46%	36%
f_9	0%	2%	70%	20%	32%	(78%)
f_{13}	36%	26%	40%	26%	26%	(50%)
f_{15}	2%	12%	38%	22%	60%	(66%)
f_{16}	4%	8%	64%	8%	34%	(84%)
f_{17}	40%	(50%)	30%	30%	30%	20%
f_{22}	26%	32%	56%	(64%)	18%	4%
f_{23}	64%	42%	24%	(56%)	12%	2%

Table 6: DE/rand/1/bin, $D = 10$, Gingerbread man (G) vs. gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	68%	(64%)	14%	30%	22%	20%
f_5	14%	10%	72%	(74%)	22%	20%
f_9	0%	0%	88%	(96%)	14%	4%
f_{13}	24%	30%	48%	(44%)	28%	26%
f_{15}	4%	8%	92%	(84%)	4%	8%
f_{16}	4%	2%	94%	(88%)	2%	10%
f_{17}	60%	(76%)	16%	4%	24%	20%
f_{22}	92%	(94%)	0%	2%	8%	4%
f_{23}	96%	(96%)	2%	2%	2%	2%

Table 7: DE/best/1/bin, $D = 20$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	100%	(90%)	0%	10%	0%	0%
f_5	98%	(90%)	2%	10%	0%	0%
f_9	0%	0%	74%	2%	28%	(100%)
f_{13}	18%	32%	38%	34%	46%	(36%)
f_{15}	6%	2%	50%	26%	48%	(72%)
f_{16}	2%	2%	72%	8%	28%	(90%)
f_{17}	88%	(70%)	12%	30%	0%	0%
f_{22}	78%	(50%)	18%	48%	4%	2%
f_{23}	88%	(54%)	12%	46%	0%	0%

Table 8: DE/best/1/bin, $D = 20$, Gingerbread man (G) vs. gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	62%	(100%)	0%	0%	38%	0%
f_5	88%	(100%)	0%	0%	12%	0%
f_9	0%	0%	98%	(100%)	2%	0%
f_{13}	22%	18%	52%	(54%)	26%	30%
f_{15}	0%	2%	80%	(82%)	26%	16%
f_{16}	4%	4%	86%	(94%)	10%	2%
f_{17}	68%	32%	0%	4%	32%	(64%)
f_{22}	62%	20%	6%	6%	32%	(74%)
f_{23}	88%	10%	6%	0%	38%	(90%)

Table 9: DE/rand/1/bin, $D = 20$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	0%	0%	60%	8%	42%	(92%)
f_5	0%	0%	66%	2%	38%	(98%)
f_9	0%	0%	6%	0%	94%	(100%)
f_{13}	32%	(40%)	32%	22%	38%	38%
f_{15}	4%	10%	60%	14%	38%	(76%)
f_{16}	6%	10%	66%	16%	28%	(74%)
f_{17}	30%	34%	38%	(36%)	32%	30%
f_{22}	48%	42%	40%	(50%)	12%	8%
f_{23}	36%	30%	54%	(70%)	10%	0%

Table 10: DE/rand/1/bin, $D = 20$, Gingerbread man (G) vs. gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	76%	(52%)	24%	48%	0%	2%
f_5	8%	2%	92%	(100%)	0%	0%
f_9	0%	0%	94%	(98%)	6%	2%
f_{13}	28%	24%	36%	36%	36%	(40%)
f_{15}	6%	10%	74%	(72%)	20%	18%
f_{16}	2%	8%	84%	(80%)	14%	12%
f_{17}	68%	82%	8%	8%	24%	10%
f_{22}	66%	(90%)	2%	2%	32%	8%
f_{23}	68%	(100%)	6%	0%	26%	0%

Table 11: DE/best/1/bin, $D = 30$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	98%	(96%)	2%	0%	0%	0%
f_5	98%	(96%)	2%	0%	0%	0%
f_9	0%	0%	54%	0%	46%	(100%)
f_{13}	32%	24%	42%	32%	28%	(46%)
f_{15}	6%	2%	42%	12%	54%	(90%)
f_{16}	4%	0%	66%	6%	32%	96%
f_{17}	96%	(88%)	4%	12%	100%	0%
f_{22}	100%	(68%)	0%	32%	48%	0%
f_{23}	98%	(64%)	2%	36%	0%	0%

Table 12: DE/best/1/bin, $D = 30$, Gingerbread man (G) vs. gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	26%	(100%)	0%	0%	74%	0%
f_5	84%	(100%)	0%	0%	16%	0%
f_9	0%	0%	100%	(100%)	0%	0%
f_{13}	22%	14%	48%	(54%)	32%	36%
f_{15}	0%	4%	94%	(88%)	6%	8%
f_{16}	2%	6%	94%	(92%)	4%	2%
f_{17}	0%	26%	0%	2%	100%	(72%)
f_{22}	50%	12%	2%	2%	48%	(86%)
f_{23}	98%	18%	4%	4%	58%	(78%)

Table 13: DE/rand/1/bin, $D = 30$, Tinkerbell (T) vs. tMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	0%	0%	52%	0%	48%	(100%)
f_5	0%	0%	68%	0%	32%	(100%)
f_9	0%	0%	74%	0%	28%	(100%)
f_{13}	22%	18%	46%	32%	34%	(50%)
f_{15}	12%	18%	64%	24%	24%	(58%)
f_{16}	16%	8%	54%	28%	30%	(64%)
f_{17}	6%	34%	48%	28%	46%	(38%)
f_{22}	54%	(92%)	0%	2%	46%	6%
f_{23}	44%	(92%)	2%	2%	54%	6%

Table 14: DE/rand/1/bin, $D = 30$, Gingerbread man (G) gMT.

	Atan2		Bounds		Modulo	
	T	tMT	T	tMT	T	tMT
f_1	0%	0%	100%	(100%)	0%	0%
f_5	0%	0%	100%	(100%)	0%	0%
f_9	0%	0%	100%	(100%)	0%	0%
f_{13}	38%	30%	38%	(38%)	26%	32%
f_{15}	14%	10%	84%	(70%)	2%	20%
f_{16}	16%	18%	74%	(64%)	10%	18%
f_{17}	20%	(76%)	12%	18%	68%	6%
f_{22}	54%	(92%)	0%	2%	46%	6%
f_{23}	44%	(92%)	2%	2%	54%	6%

Table 15: Results of the test ANOVA. The significance level has been chosen to be $\alpha = 0.1$. The highlighted values mean that in these cases the means can be considered as equal.

Algorithm	cPRNG	p-value	F crit.
DE/best/1/bin	Tink.	0.002	2.372
	Ging.	0.293	2.372
DE/rand/1/bin	Tink.	0.000	2.372
DE/best/1/bin	gMT	0.610	2.372

Table 16: Results of the T-tests for cPRNGs. The significance level has been chosen to be $\alpha = 0.1$. If the p-value is smaller than the significance level value, the null hypothesis is accepted in the case of one-sided T-test. In the case of the two-sided T-test the null hypothesis is accepted if the p-value is greater than the significance level value.

Algorithm	cPRNG	μ_A vs. μ_B (p-val.)	μ_A vs. μ_M (p-val.)	μ_B vs. μ_M (p-val.)	Resume
DE/best	Tink.	$\mu_A > \mu_B$ (0.006)	$\mu_A > \mu_M$ (0.001)	$\mu_B = \mu_M$ (0.440)	Acc.
DE/rand	Tink.	$\mu_A < \mu_B$ (0.000)	$\mu_A > \mu_M$ (0.005)	$\mu_B > \mu_M$ (0.000)	Acc.
DE/rand	Ging.	$\mu_A < \mu_B$ (0.035)	$\mu_A > \mu_M$ (0.025)	$\mu_B > \mu_M$ (0.000)	Acc.

Table 17: Results of the T-tests for MT with modified PDF. The significance level has been chosen to be $\alpha = 0.1$. If the p-value is smaller than the significance level value, the null hypothesis is accepted in the case of one-sided T-test. In the case of the two-sided T-test the null hypothesis is accepted if the p-value is greater than the significance level value.

Algorithm	cPRNG	μ_A vs. μ_B (p-val.)	μ_A vs. μ_M (p-val.)	μ_B vs. μ_M (p-val.)	Resume
DE/best	tMT	$\mu_A^{MT} > \mu_B^{MT}$ (0.003)	$\mu_A^{MT} > \mu_M^{MT}$ (0.158)	$\mu_B^{MT} = \mu_M^{MT}$ (0.209)	Acc.
DE/rand	tMT	$\mu_A^{MT} < \mu_B^{MT}$ (0.056)	$\mu_A^{MT} < \mu_M^{MT}$ (0.000)	$\mu_B^{MT} < \mu_M^{MT}$ (0.005)	Acc.
DE/rand	gMT	$\mu_A^{MT} = \mu_B^{MT}$ (0.304)	$\mu_A^{MT} > \mu_M^{MT}$ (0.001)	$\mu_B^{MT} > \mu_M^{MT}$ (0.000)	Acc.

Table 18: The comparison of the best results of the cPRNGs and MT with the modified PDF. The significance level has been chosen to be $\alpha = 0.1$. F crit. value equals to 1.675 in the one-sided test and 1.298 in the two-sided test (t_2). If the p-value is smaller than the significance level value, the null hypothesis is accepted in the case of one-sided T-test. In the case of the two-sided T-test the null hypothesis is accepted if the p-value is greater than the significance level value.

Algorithm	Comparison (norm. scheme)	Null hyp.	p-val.	Resume
DE/best	Tink. vs. tMT (Atan2)	$\mu_A = \mu_A^{tMT}$	0.361 (t_2)	Acc.
DE/best	Ging. vs. gMT (Atan2)	$\mu_A = \mu_A^{gMT}$	0.544 (t_2)	Acc.
	Ging. vs. gMT (Bounds)	$\mu_B = \mu_B^{gMT}$	0.944 (t_2)	Acc.
	Ging. vs. gMT (Modulo)	$\mu_M = \mu_M^{gMT}$	0.776 (t_2)	Acc.
DE/rand	Tink. vs. tMT (Atan2)	$\mu_A = \mu_A^{tMT}$	0.530 (t_2)	Acc.
	Tink. vs. tMT (Bounds)	$\mu_B > \mu_B^{tMT}$	0.000	Acc.
	Tink. vs. tMT (Modulo)	$\mu_M < \mu_M^{tMT}$	0.010	Acc.
	Tink. vs. tMT (Bounds/Modulo)	$\mu_B = \mu_M^{tMT}$	0.809 (t_2)	Acc.
DE/rand	Ging. vs gMT (Atan2)	$\mu_A = \mu_A^{gMT}$	0.455 (t_2)	Acc.
DE/rand	Ging. vs. gMT (Bounds)	$\mu_B = \mu_B^{gMT}$	0.983 (t_2)	Acc.

Differential Evolution Powered by Chaos Using Three Types of Number Normalization – Complementary Materials Part I

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Abstract. In many publications, authors showed that chaotic pseudo random numbers generators (PRNGs) may improve performance of the evolutionary algorithms. In this paper, we use two chaotic maps Gingerbread man and Tinkerbell as the chaotic PRNGs instead of the classical PRNG in the algorithm differential evolution. Numbers generated by this maps are normalized to the interval $[0,1]$ by three different methods – operation modulo, classical normalization and function atan2 . The goal is to show that number normalization may affect the differential evolution convergence speed. In the second part of this paper, we have modified well known PRNG Mersenne Twister to generate numbers with the same probability distribution function like the chaotic maps using different schemes of number normalization, where we have tested if the PRNG with the same probability distribution function will reach the same results. The results mentioned below showed that the selected normalization method may improve differential evolution convergence speed, especially in the case of function atan2 and classical normalization. Modified Mersenne Twister reached very different results than chaotic PRNGs with the different number normalization. That means the distribution of the selected (chaotic) PRNG with the number normalization scheme alone does not suffice to improve differential evolution convergence speed and play the key role in this process.

* The following grants are acknowledged for the financial support provided for this research: Grant Agency of the Czech Republic - GACR P103/13/08195S, is partially supported by Grant of SGS No. SP2014/42, VB - Technical University of Ostrava, Czech Republic, by the Development of human resources in research and development of latest soft computing methods and their application in practice project, reg. no. CZ.1.07/2.3.00/20.0072 funded by Operational Programme Education for Competitiveness.

Keywords: Pseudo random number generator, evolutionary algorithms, differential evolution, particle swarm, chaos, Gingerbread man, Tinkerbell

1 Introduction

In this materials, we have mentioned statistic analysis for paper *Differential Evolution Powered by Chaos Using Three Types of Number Normalization*. Materials have been divided into two parts. In this part, statistic analysis of differential evolution (DE) using chaotic pseudo random number generators (cPRNGs) – Gingerbread man map and Tinkerbell map with three types of number normalization are described. In the tables below we are comparing results of the used number normalization schemes: Atan2, classical normalization denoted as Bounds and operator Modulo. The tables are divided into sections according to the dimension. Experiments for dimensions $D = 10$, $D = 20$ and $D = 30$ have been executed. Each experiment has been repeated fifty times.

2 Results

Table 1: DE/best/1/bin, $D = 10$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.	
f_1	Atan2	-1400.000	-1341.777	-1398.587	-1400.000	8.170	
	Bounds	-1116.067	5993.978	1496.679	1116.439	1559.007	
	Modulo	-1400.000	-1297.048	-1394.534	-1399.9	17.829	
f_5	Atan2	-100.000	0	-994.681	-999.676	-999.976	0.873
	Bounds	-982.066	7010.481	57.870	-615.200	1484.354	
	Modulo	-999.986	-843.512	-974.057	-995.859	40.033	
f_9	Atan2	-599.997	-599.796	-599.915	-599.914	0.053	
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000	
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000	
f_{13}	Atan2	-200.000	-199.995	-200.000	-200.000	0.001	
	Bounds	-200.000	-199.995	-200.000	-200.000	0.001	
	Modulo	-200.000	-199.999	-200.000	-200.000	0.000	
f_{15}	Atan2	100.000	100.079	100.008	100.003	0.013	
	Bounds	100.000	100.625	100.175	100.000	0.281	
	Modulo	100.000	100.625	100.062	100.000	0.188	
f_{16}	Atan2	200.001	200.033	200.010	200.007	0.008	
	Bounds	200.000	200.000	200.000	200.000	0.000	
	Modulo	200.000	200.000	200.000	200.000	0.000	
f_{17}	Atan2	304.742	352.964	325.632	325.031	8.609	
	Bounds	319.947	410.029	354.933	349.04	23.031	
	Modulo	314.153	343.286	327.063	325.756	7.343	
f_{22}	Atan2	805.251	1222.601	923.846	927.392	113.616	
	Bounds	1135.939	2310.04	1589.834	1535.226	257.559	
	Modulo	851.631	2352.702	1228.477	1178.923	271.302	
f_{23}	Atan2	901.469	1272.474	1022.004	1027.407	98.91	
	Bounds	1096.909	2555.113	1747.235	1719.500	337.116	
	Modulo	929.746	1652.331	1261.346	1253.039	160.744	

Table 2: DE/best/1/bin, $D = 10$, Tinkerbell as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1400.000	-841.201	-1359.348	-1398.399	98.545
	Bounds	-1399.944	-529.141	-1282.48	-1343.21	162.301
	Modulo	-1389.095	1943.356	-667.012	-984.147	829.98
f_5	Atan2	-999.933	-610.199	-979.291	-994.797	55.478
	Bounds	-999.179	256.076	-730.748	-822.204	245.909
	Modulo	-976.834	1712.885	-570.704	-843.251	558.92
f_9	Atan2	-600.000	-600.000	-600.000	-600.000	0.000
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-199.998	-200.000	-200.000	0.000
	Modulo	-200.000	-199.988	-200.000	-200.000	0.002
f_{15}	Atan2	100.000	100.625	100.038	100.000	0.148
	Bounds	-477.635	556.689	126.572	100.000	235.498
	Modulo	100.000	133.525	100.883	100.000	4.672
f_{16}	Atan2	200.000	200.000	200.000	200.000	0.000
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	308.447	363.751	329.199	324.753	11.754
	Bounds	323.863	378.764	346.671	343.952	13.160
	Modulo	315.439	480.962	373.148	368.189	38.656
f_{22}	Atan2	837.185	1854.352	1264.432	1272.591	221.900
	Bounds	998.865	2214.721	1618.634	1589.480	275.603
	Modulo	1214.059	2301.915	1779.788	1762.625	216.925
f_{23}	Atan2	923.015	1799.224	1340.532	1330.394	198.328
	Bounds	1101.238	2290.375	1647.917	1620.352	251.598
	Modulo	1238.201	2539.776	1840.882	1796.526	303.993

Table 3: DE/rand1/bin, $D = 10$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Modulo	-1400.000	-1400.000	-1400.000	-1400.000	0.000
f_5	Atan2	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Bounds	-1000.000	-999.958	-999.999	-1000.000	0.006
	Modulo	-1000.000	-999.999	-1000.000	-1000.000	0.000
f_9	Atan2	-599.99	-599.709	-599.904	-599.918	0.062
	Bounds	-600.000	-599.963	-599.998	-600.000	0.006
	Modulo	-600.000	-599.953	-599.988	-599.994	0.013
f_{13}	Atan2	-200.000	-199.988	-199.999	-200.000	0.002
	Bounds	-200.000	-199.999	-200.000	-200.000	0.000
	Modulo	-200.000	-199.997	-200.000	-200.000	0.001
f_{15}	Atan2	101.521	100.210	100.854	100.008	1.191
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.037	200.009	200.006	0.009
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	325.563	348.265	337.237	336.928	5.163
	Bounds	328.465	359.193	346.032	346.937	6.573
	Modulo	327.267	351.456	341.335	341.130	5.663
f_{22}	Atan2	1426.673	2136.490	1820.351	1819.956	162.442
	Bounds	1931.084	2821.204	2476.900	2525.028	192.040
	Modulo	1650.227	2504.549	2173.053	2220.269	200.025
f_{23}	Atan2	1436.007	2249.422	1902.986	1871.902	184.527
	Bounds	2036.26	2863.141	2558.708	2599.453	188.074
	Modulo	2001.839	2590.124	2313.22	2315.500	137.794

Table 4: DE/rand1/bin, $D = 10$, Tinkerbell as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1400.000	-1399.998	-1400.000	-1400.000	0.000
	Modulo	-1400.000	-1400.000	-1400.000	-1400.000	0.000
f_5	Atan2	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Bounds	-1000.000	-999.980	-999.999	-1000.000	0.003
	Modulo	-1000.000	-999.990	-1000.000	-1000.000	0.002
f_9	Atan2	-599.997	-599.832	-599.956	-599.963	0.035
	Bounds	-600.000	-599.999	-600.000	-600.000	0.000
	Modulo	-600.000	-599.982	-599.999	-600.000	0.003
f_{13}	Atan2	-200.000	-199.997	-200.000	-200.000	0.001
	Bounds	-200.000	-199.999	-200.000	-200.000	0.000
	Modulo	-200.000	-199.999	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.002	100.000	100.000	0.000
	Bounds	100.046	100.000	100.619	100.000	35.666
	Modulo	100.000	100.625	100.012	100.000	0.088
f_{16}	Atan2	200.000	200.003	200.000	200.000	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	332.735	355.353	345.564	345.569	5.097
	Bounds	329.598	357.027	346.826	347.534	5.890
	Modulo	334.358	357.335	346.535	346.862	4.903
f_{22}	Atan2	1904.675	2773.616	2397.492	2399.587	180.575
	Bounds	1899.416	2653.072	2347.224	2382.455	173.750
	Modulo	2235.293	2981.175	2560.031	2547.184	184.711
f_{23}	Atan2	1999.500	2747.983	2465.606	2488.092	161.808
	Bounds	2111.503	2801.723	2555.803	2575.176	164.352
	Modulo	2339.204	3019.777	2743.248	2779.833	174.702

Table 5: DE/best/1/bin, $D = 20$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1387.604	-1100.093	-1316.103	-1342.547	67.566
	Bounds	1902.179	16639.775	7386.17	6686.369	3103.2
	Modulo	-1387.33	-578.033	-1255.855	-1329.492	157.17
f_5	Atan2	-996.984	-595.301	-955.837	-978.760	70.783
	Bounds	396.226	26807.775	9708.291	7687.590	6089.540
	Modulo	-988.63	-638.202	-886.868	-913.376	85.447
f_9	Atan2	-599.997	-599.822	-599.954	-599.961	0.031
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.012	100.001	100.000	0.003
	Bounds	100.000	100.625	100.112	100.000	0.240
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.007	200.002	200.001	0.002
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	341.949	460.559	371.627	366.605	20.907
	Bounds	421.263	627.999	517.351	516.540	45.591
	Modulo	346.566	432.123	382.210	380.366	21.349
f_{22}	Atan2	1037.318	4322.624	2057.094	1424.061	1010.075
	Bounds	1953.134	3966.631	2963.150	2925.088	389.365
	Modulo	1140.189	4663.335	2118.285	1812.806	819.974
f_{23}	Atan2	1100.186	4343.556	2063.710	1529.934	956.181
	Bounds	1997.346	4010.445	3080.884	3120.196	454.936
	Modulo	1281.007	4860.38	2250.993	1995.202	757.254

Table 6: DE/best/1/bin, $D = 20$, Tinkerbell as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1352.931	463.874	-929.445	-1066.077	397.890
	Bounds	-155.174	8600.879	3948.886	3865.264	1980.206
	Modulo	722.882	21308.813	7738.186	7603.403	4155.355
f_5	Atan2	-947.914	-201.343	-719.583	-762.926	157.353
	Bounds	-589.16	2915.649	202.711	-52.830	704.195
	Modulo	-302.481	10984.309	2686.347	2167.904	2314.304
f_9	Atan2	-600.000	-599.994	-599.999	-600.000	0.002
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.000	100.000	100.000	0.000
	Bounds	100.000	100.625	100.050	100.000	0.170
	Modulo	100.000	100.625	100.038	100.000	0.148
f_{16}	Atan2	200.000	200.002	200.000	200.000	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	356.858	468.409	404.542	405.501	28.43
	Bounds	392.308	617.557	497.612	496.311	51.246
	Modulo	473.335	802.701	624.941	630.126	79.226
f_{22}	Atan2	1104.518	2916.903	2202.173	2174.775	419.821
	Bounds	1834.879	4090.583	2832.729	2839.605	502.152
	Modulo	2605.661	4889.415	3579.584	3478.275	524.926
f_{23}	Atan2	1465.539	3550.791	2294.342	2214.173	420.123
	Bounds	2107.927	3864.456	2985.016	2876.596	431.398
	Modulo	2879.702	4476.526	3688.043	3635.495	411.111

Table 7: DE/rand/1/bin, $D = 20$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1400.000	-1399.999	-1400.000	-1400.000	0.000
	Modulo	-1399.999	-1399.996	-1399.998	-1399.998	0.001
f_5	Atan2	-999.998	-999.991	-999.996	-999.996	0.002
	Bounds	-999.999	-999.996	-999.998	-999.999	0.001
	Modulo	-999.997	-999.977	-999.989	-999.990	0.003
f_9	Atan2	-599.989	-599.667	-599.917	-599.920	0.058
	Bounds	-600.000	-599.999	-600.000	-600.000	0.000
	Modulo	-600.000	-599.933	-599.987	-599.996	0.018
f_{13}	Atan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-199.999	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.031	100.003	100.001	0.005
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.008	200.002	200.002	0.002
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	379.258	432.549	415.569	416.546	9.998
	Bounds	413.537	452.665	434.284	435.89	8.986
	Modulo	406.75	439.744	424.096	423.655	7.271
f_{22}	Atan2	3925.356	4953.557	4531.137	4588.548	244.91
	Bounds	4652.821	5848.462	5305.086	5365.198	275.639
	Modulo	4149.028	5181.361	4712.121	4682.958	233.926
f_{23}	Atan2	4047.685	5003.567	4648.810	4629.876	218.292
	Bounds	4199.932	5775.777	5370.133	5427.54	306.362
	Modulo	4424.592	5259.613	4858.645	4834.817	220.416

Table 8: DE/rand/1/bin, $D = 20$, Tinkerbell as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1400.000	-1399.995	-1399.998	-1399.999	0.001
	Bounds	-1400.000	-1399.999	-1400.000	-1400.000	0.000
	Modulo	-1400.000	-1399.998	-1400.000	-1400.000	0.000
f_5	Atan2	-999.997	-999.978	-999.993	-999.993	0.004
	Bounds	-1000.000	-999.996	-999.999	-999.999	0.001
	Modulo	-1000.000	-999.983	-999.998	-999.998	0.002
f_9	Atan2	-599.999	-599.824	-599.928	-599.923	0.041
	Bounds	-600.000	-599.976	-599.998	-600.000	0.004
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.007	100.001	100.000	0.001
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.014	200.002	200.001	0.002
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	414.185	450.125	434.119	434.324	7.794
	Bounds	412.750	452.961	432.212	431.821	8.486
	Modulo	410.002	448.263	432.205	432.986	9.736
f_{22}	Atan2	4320.667	5572.205	5149.487	5182.842	256.646
	Bounds	4796.348	5578.235	5265.008	5291.438	179.715
	Modulo	4579.861	5968.813	5529.692	5574.073	269.622
f_{23}	Atan2	4906.254	5684.771	5291.404	5307.882	188.15
	Bounds	4475.200	5698.890	5262.358	5280.326	253.094
	Modulo	4954.935	6136.736	5636.929	5628.762	240.277

Table 9: DE/best/1/bin, $D = 30$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1160.086	12716.359	3013.109	1545.624	3605.074
	Bounds	20203.914	34264.46	27833.000	27788.314	3594.681
	Modulo	-949.447	2001.225	104.444	-49.506	728.074
f_5	Atan2	-961.867	-314.609	-793.431	-845.34	149.438
	Bounds	5266.778	22238.48	10808.168	10527.986	3397.012
	Modulo	-931.488	1014.898	-486.132	-614.622	339.51
f_9	Atan2	-599.999	-599.868	-599.956	-599.966	0.035
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.003	100.000	100.000	0.001
	Bounds	100.000	100.625	100.025	100.000	0.122
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.004	200.001	200.001	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	577.644	1334.903	881.800	898.120	185.557
	Bounds	587.407	957.434	801.326	806.712	79.864
	Modulo	401.269	585.257	483.763	485.419	45.209
f_{22}	Atan2	1717.652	6795.64	3245.242	2424.778	1515.202
	Bounds	3298.243	6544.876	4896.187	4892.618	684.950
	Modulo	1540.765	7130.642	3637.302	2571.576	1909.083
f_{23}	Atan2	1837.621	7430.865	3766.299	3156.175	1491.173
	Bounds	3364.503	6424.378	4841.783	4882.867	593.313
	Modulo	1567.382	7316.444	3628.83	2763.922	1792.710

Table 10: DE/best/1/bin, $D = 30$, Tinkerbelle as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-840.274	6418.535	1872.388	1667.874	1658.737
	Bounds	3972.713	16770.832	10629.256	10333.05	3371.281
	Modulo	5183.542	30649.726	15816.771	15680.372	5425.529
f_5	Atan2	-756.497	2387.973	94.531	-107.402	682.553
	Bounds	230.041	15681.528	5011.468	4656.675	2984.968
	Modulo	1526.782	16071.356	5735.504	4536.36	3642.97
f_9	Atan2	-600.000	-599.938	-599.988	-599.991	0.011
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.001	100.000	100.000	0.000
	Bounds	100.000	100.625	100.012	100.000	0.088
	Modulo	100.000	100.625	100.025	100.000	0.122
f_{16}	Atan2	200.000	200.002	200.001	200.000	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	421.242	697.031	532.944	523.608	53.718
	Bounds	541.762	841.093	684.508	678.06	76.899
	Modulo	725.434	1503.405	978.312	959.139	161.001
f_{22}	Atan2	2465.905	4769.877	3529.185	3459.975	495.186
	Bounds	3657.473	6140.259	4830.445	4838.858	501.624
	Modulo	4003.421	6939.121	5323.996	5124.838	609.699
f_{23}	Atan2	2462.798	4692.169	3518.939	3449.835	585.721
	Bounds	3752.254	6105.143	4900.956	4899.206	508.689
	Modulo	3848.027	7042.543	5385.929	5319.222	551.583

Table 11: DE/rand/1/bin, $D = 30$, Gingerbread man as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1383.087	-1255.424	-1338.028	-1338.288	29.024
	Bounds	-1399.999	-1399.994	-1399.997	-1399.997	0.001
	Modulo	-1399.976	-1399.858	-1399.937	-1399.942	0.027
f_5	Atan2	-996.31	-984.081	-991.419	-991.418	2.811
	Bounds	-999.996	-999.968	-999.987	-999.989	0.006
	Modulo	-999.893	-999.612	-999.784	-999.799	0.063
f_9	Atan2	-599.993	-599.860	-599.939	-599.942	0.037
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-599.999	-599.946	-599.992	-599.996	0.009
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.010	100.001	100.001	0.002
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.001	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.004	200.001	200.001	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.004	200.001	200.000	0.001
f_{17}	Atan2	503.214	563.042	534.203	534.078	13.003
	Bounds	502.647	553.711	532.747	533.421	10.599
	Modulo	492.116	545.349	521.323	522.178	10.818
f_{22}	Atan2	6447.57	7812.904	7360.456	7421.433	303.05
	Bounds	7670.873	8753.254	8290.195	8319.742	261.850
	Modulo	6480.763	7978.502	7407.955	7450.165	311.339
f_{23}	Atan2	6759.360	7908.918	7524.860	7531.540	214.546
	Bounds	7207.221	8830.549	8376.995	8388.011	298.475
	Modulo	6693.331	8072.473	7463.749	7488.474	312.722

Table 12: DE/rand/1/bin, $D = 30$, Tinkerbell as the cPRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Atan2	-1399.984	-1399.929	-1399.963	-1399.964	0.013
	Bounds	-1399.999	-1399.995	-1399.998	-1399.998	0.001
	Modulo	-1399.999	-1399.993	-1399.997	-1399.998	0.001
f_5	Atan2	-999.959	-999.747	-999.895	-999.899	0.035
	Bounds	-999.997	-999.972	-999.991	-999.993	0.005
	Modulo	-999.995	-999.97	-999.989	-999.990	0.005
f_9	Atan2	-599.996	-599.862	-599.955	-599.968	0.036
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Atan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Atan2	100.000	100.013	100.001	100.001	0.002
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Atan2	200.000	200.003	200.001	200.001	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Atan2	521.051	553.329	537.329	536.29	6.938
	Bounds	505.289	556.041	529.966	530.000	11.116
	Modulo	501.734	551.515	527.512	527.562	12.554
f_{22}	Atan2	7410.675	8609.496	8142.026	8146.476	275.253
	Bounds	7317.685	8616.992	8159.274	8188.075	257.534
	Modulo	7681.315	9005.522	8532.965	8551.819	315.234
f_{23}	Atan2	7114.605	8734.946	8237.983	8206.852	320.252
	Bounds	6973.415	8688.827	8247.023	8282.936	319.869
	Modulo	7826.880	9210.351	8671.689	8701.908	316.752

Differential Evolution Powered by Chaos Using Three Types of Number Normalization – Complementary Materials Part II

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Abstract. In many publications, authors showed that chaotic pseudo random numbers generators (PRNGs) may improve performance of the evolutionary algorithms. In this paper, we use two chaotic maps Gingerbread man and Tinkerbell as the chaotic PRNGs instead of the classical PRNG in the algorithm differential evolution. Numbers generated by this maps are normalized to the interval $[0,1]$ by three different methods – operation modulo, classical normalization and function atan2 . The goal is to show that number normalization may affect the differential evolution convergence speed. In the second part of this paper, we have modified well known PRNG Mersenne Twister to generate numbers with the same probability distribution function like the chaotic maps using different schemes of number normalization, where we have tested if the PRNG with the same probability distribution function will reach the same results. The results mentioned below showed that the selected normalization method may improve differential evolution convergence speed, especially in the case of function atan2 and classical normalization. Modified Mersenne Twister reached very different results than chaotic PRNGs with the different number normalization. That means the distribution of the selected (chaotic) PRNG with the number normalization scheme alone does not suffice to improve differential evolution convergence speed and play the key role in this process.

* The following grants are acknowledged for the financial support provided for this research: Grant Agency of the Czech Republic - GACR P103/13/08195S, is partially supported by Grant of SGS No. SP2014/42, VB - Technical University of Ostrava, Czech Republic, by the Development of human resources in research and development of latest soft computing methods and their application in practice project, reg. no. CZ.1.07/2.3.00/20.0072 funded by Operational Programme Education for Competitiveness.

Keywords: Pseudo random number generator, evolutionary algorithms, differential evolution, particle swarm, chaos, Gingerbread man, Tinkerbelle

1 Introduction

In this materials, we have mentioned statistic analysis for paper *Differential Evolution Powered by Chaos Using Three Types of Number Normalization*. Materials have been divided into two parts. In this part statistic analysis of differential evolution (DE) using Mersenne Twister (MT) modified to generate number with the same probability as the chaotic pseudo random number generators using different schemes of number normalization: Arctan2, classical normalization denoted as Bounds and operator Modulo. The tables are divided into sections according to the dimension. Experiments for dimensions $D = 10$, $D = 20$ and $D = 30$ have been executed. Each experiment has been repeated fifty times.

2 Results

Table 1: DE/best/1/bin, $D = 10$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1399.998	-543.168	-1260.622	-1329.027	179.961
	Modulo	-1400.000	-1367.996	-1399.200	-1400.000	4.496
f_5	Arctan2	-1000.000	-999.998	-1000.000	-1000.000	0.001
	Bounds	-999.666	-228.171	-899.000	-931.948	130.020
	Modulo	-999.999	-965.944	-996.079	-999.742	7.650
f_9	Arctan2	-599.999	-599.860	-599.957	-599.962	0.033
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-199.997	-200.000	-200.000	0.001
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.014	110.015	100.999	100.000	6.960
	Bounds	100.000	100.625	100.100	100.000	0.229
	Modulo	100.000	100.625	100.250	100.000	0.122
f_{16}	Arctan2	200.000	200.015	200.004	200.002	0.004
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	309.892	338.606	323.754	322.978	5.190
	Bounds	316.358	367.412	339.247	339.643	12.210
	Modulo	313.711	357.876	331.997	330.887	10.127
f_{22}	Arctan2	800.847	1747.017	1076.103	974.204	245.46
	Bounds	1038.462	2246.641	1500.627	1442.897	290.017
	Modulo	837.852	2405.108	1163.750	1094.621	242.623
f_{23}	Arctan2	900.583	2126.449	1147.414	1081.682	278.586
	Bounds	1061.44	2060.152	1583.094	1571.790	270.48
	Modulo	904.552	2557.851	1238.541	1171.907	256.142

Table 2: DE/best/1/bin, $D = 10$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1298.078	-1394.962	-1399.930	16.501
	Bounds	-1400.000	-1093.382	-1381.411	-1398.631	48.345
	Modulo	-1386.506	3875.369	-445.359	-875.620	1073.647
f_5	Arctan2	-999.987	-933.628	-992.378	-998.746	13.799
	Bounds	-999.974	-632.814	-945.143	-970.713	66.124
	Modulo	-989.827	7327.729	-239.565	-755.257	1274.47
f_9	Arctan2	-600.000	-600.000	-600.000	-600.000	0.000
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-199.998	-200.000	-200.000	0.000
	Bounds	-200.000	-199.999	-200.000	-200.000	0.000
	Modulo	-200.000	-199.998	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.000	100.000	100.000	0.000
	Bounds	100.000	556.689	276.994	100.000	8.762
	Modulo	100.000	100.625	100.138	100.000	0.259
f_{16}	Arctan2	200	200.000	200.000	200.000	0.000
	Bounds	200	200.000	200.000	200.000	0.000
	Modulo	200	200.000	200.000	200.000	0.000
f_{17}	Arctan2	313.274	346.981	328.183	326.316	8.758
	Bounds	308.661	361.766	328.583	326.266	10.495
	Modulo	316.000	513.858	376.242	364.300	41.173
f_{22}	Arctan2	809.493	1534.713	1111.826	1082.406	184.123
	Bounds	825.09	1750.738	1220.160	1213.822	212.871
	Modulo	1411.634	2740.268	1912.244	1890.673	277.341
f_{23}	Arctan2	937.77	1916.279	1266.210	1252.041	189.639
	Bounds	934.528	1897.399	1305.578	1312.610	207.992
	Modulo	1306.41	2913.450	2037.080	2020.470	331.135

Table 3: DE/rand/1/bin, $D = 10$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Modulo	-1400.000	-1400.000	-1400.000	-1400.000	0.000
f_5	Arctan2	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Bounds	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Modulo	-1000.000	-1000.000	-1000.000	-1000.000	0.000
f_9	Arctan2	-599.969	-599.645	-599.900	-599.911	0.057
	Bounds	-600.000	-599.977	-599.997	-599.999	0.006
	Modulo	-599.995	-599.857	-599.958	-599.962	0.028
f_{13}	Arctan2	-200.000	-199.995	-199.999	-200.000	0.001
	Bounds	-200.000	-199.999	-200.000	-200.000	0.000
	Modulo	-200.000	-199.997	-200.000	-200.000	0.001
f_{15}	Arctan2	100.475	100.421	100.125	100.005	41.797
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.492	100.001	100.470	100.000	24.709
f_{16}	Arctan2	200.000	200.021	200.006	200.003	0.006
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.004	200.000	200.000	0.001
f_{17}	Arctan2	325.492	342.398	334.830	334.929	4.447
	Bounds	333.656	357.437	345.634	346.046	5.539
	Modulo	327.895	353.500	342.188	341.58	5.551
f_{22}	Arctan2	1457.737	2230.297	1939.686	1953.902	152.124
	Bounds	1944.26	2769.098	2454.918	2502.788	211.347
	Modulo	1812.063	2519.411	2266.318	2296.410	145.249
f_{23}	Arctan2	1640.755	2330.858	2049.640	2060.050	146.806
	Bounds	1911.238	2869.264	2573.845	2586.203	176.245
	Modulo	1768.039	2690.639	2375.162	2401.844	179.392

Table 4: DE/rand/1/bin, $D = 10$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Modulo	-1400.000	-1400.000	-1400.000	-1400.000	0.000
f_5	Arctan2	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Bounds	-1000.000	-999.999	-1000.000	-1000.000	0.000
	Modulo	-1000.000	-999.732	-999.989	-1000.000	0.042
f_9	Arctan2	-600.000	-599.911	-599.971	-599.980	0.023
	Bounds	-600.000	-599.957	-599.994	-599.997	0.008
	Modulo	-600.000	-599.975	-599.999	-600.000	0.004
f_{13}	Arctan2	-200.000	-199.996	-200.000	-200.000	0.001
	Bounds	-200.000	-199.998	-200.000	-200.000	0.000
	Modulo	-200.000	-199.998	-200.000	-200.000	0.000
f_{15}	Arctan2	100.699	150.000	100.014	100.000	6.902
	Bounds	100.000	200.000	188.599	100.000	79.804
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Arctan2	200.000	200.001	200.000	200.000	0.000
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	324.804	350.083	342.01	342.916	5.775
	Bounds	327.191	357.315	344.436	344.144	5.512
	Modulo	333.213	360.103	348.124	348.872	5.581
f_{22}	Arctan2	2035.384	2582.546	2326.619	2339.946	126.429
	Bounds	1765.065	2571.528	2249.116	2267.661	174.433
	Modulo	2118.59	2876.088	2605.377	2633.97	175.759
f_{23}	Arctan2	1778.295	2712.099	2422.874	2410.294	188.646
	Bounds	1776.182	2694.803	2379.299	2414.698	181.428
	Modulo	2173.916	3077.228	2679.886	2648.368	174.391

Table 5: DE/best/1/bin, $D = 20$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1400.000	-1400.000	-1400.000	0.000
	Bounds	-1282.692	2518.693	316.022	54.982	984.982
	Modulo	-1398.115	-1075.507	-1340.343	-1369.188	74.539
f_5	Arctan2	-1000.000	-997.652	-999.834	-999.974	0.392
	Bounds	-873.216	354.622	-352.774	-451.436	321.003
	Modulo	-995.515	-565.575	-903.114	-932.396	92.185
f_9	Arctan2	-599.997	-599.844	-599.94	-599.95	0.034
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-599.995	-600.000	-600.000	0.001
f_{13}	Arctan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.023	100.002	100.000	0.004
	Bounds	100.000	100.625	100.075	100.000	0.203
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Arctan2	200.000	200.008	200.002	200.001	0.002
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.001	200.000	200.000	0.000
f_{17}	Arctan2	362.61	427.736	395.293	395.121	14.85
	Bounds	360.366	526.657	437.905	435.692	36.708
	Modulo	347.367	443.856	383.256	377.338	21.917
f_{22}	Arctan2	1011.458	4743.994	3168.57	3523.996	1072.429
	Bounds	2083.091	4002.271	2767.88	2631.312	425.664
	Modulo	1180.729	4237.154	1991.982	1849.752	590.778
f_{23}	Arctan2	931.870	4563.447	3412.177	3656.390	863.717
	Bounds	2227.591	4267.887	3060.492	3086.367	395.678
	Modulo	1096.715	3615.703	1941.779	1905.285	440.927

Table 6: DE/best/1/bin, $D = 20$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1398.036	-134.889	-1257.505	-1333.242	215.403
	Bounds	-1361.436	1143.671	-718.070	-834.218	521.281
	Modulo	35.935	16206.634	6759.4	5539.946	4145.321
f_5	Arctan2	-991.205	-483.793	-856.874	-884.517	105.592
	Bounds	-930.585	254.411	-607.085	-676.418	253.94
	Modulo	-384.271	14903.879	2441.680	1148.900	3054.176
f_9	Arctan2	-600.000	-600.000	-600.000	-600.000	0.000
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.000	100.000	100.000	0.000
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.625	100.075	100.000	0.203
f_{16}	Arctan2	200.000	200.000	200.000	200.000	0.000
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	352.976	449.425	386.082	381.857	21.929
	Bounds	354.668	510.854	406.358	396.254	35.598
	Modulo	481.185	800.614	617.112	595.871	76.102
f_{22}	Arctan2	1365.244	4555.627	2032.856	1879.404	578.011
	Bounds	1252.806	3128.430	2013.81	1958.396	408.931
	Modulo	2352.933	4634.598	3534.455	3485.709	455.567
f_{23}	Arctan2	1296.53	3592.456	2147.816	2076.486	427.687
	Bounds	1140.951	3020.355	2164.236	2103.236	433.595
	Modulo	2358.032	4470.144	3678.072	3666.906	399.344

Table 7: DE/rand/1/bin, $D = 20$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1399.999	-1399.999	-1400.000	0.000
	Bounds	-1400.000	-1399.998	-1400.000	-1400.000	0.001
	Modulo	-1400.000	-1399.994	-1399.998	-1399.998	0.001
f_5	Arctan2	-999.996	-999.981	-999.990	-999.991	0.003
	Bounds	-999.999	-999.995	-999.998	-999.998	0.001
	Modulo	-999.996	-999.974	-999.986	-999.987	0.005
f_9	Arctan2	-599.999	-599.796	-599.902	-599.906	0.055
	Bounds	-600.000	-599.963	-599.999	-600.000	0.005
	Modulo	-599.994	-599.82	-599.927	-599.948	0.046
f_{13}	Arctan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-199.999	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.019	100.002	100.001	0.003
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.004	100.001	100.000	0.001
f_{16}	Arctan2	200.000	200.008	200.002	200.001	0.002
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.006	200.001	200.001	0.001
f_{17}	Arctan2	397.737	436.526	413.181	412.688	7.593
	Bounds	397.481	451.090	432.064	434.148	11.293
	Modulo	384.993	443.110	426.852	428.611	11.003
f_{22}	Arctan2	3745.113	4985.662	4499.735	4549.454	266.575
	Bounds	4715.875	5684.208	5299.725	5302.649	230.247
	Modulo	4223.599	5532.44	5040.932	5075.166	291.684
f_{23}	Arctan2	3787.847	5027.684	4585.461	4629.588	240.793
	Bounds	4537.231	5930.823	5374.175	5387.103	266.183
	Modulo	4687.492	5454.943	5202.772	5223.374	162.017

Table 8: DE/rand/1/bin, $D = 20$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1399.995	-1399.998	-1399.998	0.001
	Bounds	-1400.000	-1399.996	-1399.999	-1399.999	0.001
	Modulo	-1400.000	-1399.999	-1400.000	-1400.000	0.000
f_5	Arctan2	-999.997	-999.976	-999.990	-999.990	0.004
	Bounds	-999.997	-999.984	-999.993	-999.994	0.003
	Modulo	-999.999	-999.988	-999.998	-999.998	0.002
f_9	Arctan2	-599.994	-599.859	-599.949	-599.956	0.034
	Bounds	-599.999	-599.853	-599.960	-599.959	0.028
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.002	100.000	100.000	0.001
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Arctan2	200.000	200.006	200.001	200.001	0.001
	Bounds	200.000	200.001	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	406.470	448.145	433.246	434.444	9.133
	Bounds	403.830	448.244	432.673	433.968	9.864
	Modulo	408.624	448.040	432.424	433.088	10.020
f_{22}	Arctan2	4247.299	5537.476	5088.883	5106.585	240.401
	Bounds	4430.296	5373.587	5021.548	5016.996	206.209
	Modulo	4554.765	5962.846	5538.612	5565.592	263.652
f_{23}	Arctan2	4779.654	5597.920	5195.431	5198.700	189.379
	Bounds	4390.155	5529.673	5081.831	5025.076	245.398
	Modulo	5063.634	6120.389	5649.059	5657.216	230.616

Table 9: DE/best/1/bin, $D = 30$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1400.000	-1398.44	-1399.918	-1399.991	0.247
	Bounds	361.435	12408.217	4467.608	3708.208	2691.050
	Modulo	-1373.428	-201.012	-947.751	-988.584	242.055
f_5	Arctan2	-999.916	-965.972	-995.293	-997.750	6.126
	Bounds	-423.686	4044.466	1023.596	756.262	973.39
	Modulo	-910.586	411.154	-570.448	-658.427	267.369
f_9	Arctan2	-599.996	-599.757	-599.952	-599.965	0.039
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-599.963	-599.991	-599.996	0.009
f_{13}	Arctan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.003	100.001	100.000	0.001
	Bounds	100.000	100.625	100.025	100.000	0.122
	Modulo	100.000	100.001	100.000	100.000	0.000
f_{16}	Arctan2	200.000	200.008	200.001	200.001	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.002	200.001	200.000	0.001
f_{17}	Arctan2	376.895	539.508	470.779	473.626	33.890
	Bounds	450.956	693.066	571.240	572.08	54.378
	Modulo	381.426	588.314	444.764	438.941	37.937
f_{22}	Arctan2	983.044	7226.554	5506.071	6141.600	1740.493
	Bounds	2722.759	6108.857	4567.795	4482.420	717.445
	Modulo	1419.154	6524.623	2666.791	2622.364	734.294
f_{23}	Arctan2	957.404	7510.061	5145.726	5837.85	1831.336
	Bounds	3323.494	5908.219	4437.72	4370.952	541.470
	Modulo	2356.316	8730.621	3465.04	2916.074	1546.457

Table 10: DE/best/1/bin, $D = 30$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1304.879	894.048	-632.994	-767.292	492.084
	Bounds	-598.333	10487.748	2695.652	2117.127	2086.825
	Modulo	-1373.428	-201.012	-947.751	-988.584	242.055
f_5	Arctan2	-842.382	1054.71	-439.667	-534.479	340.156
	Bounds	-639.259	4973.242	737.108	526.642	1010.894
	Modulo	-18.292	23427.223	4630.305	3928.422	3471.253
f_9	Arctan2	-600.000	-599.991	-600.000	-600.000	0.001
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.000	100.000	100.000	0.000
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.625	100.012	100.000	0.088
f_{16}	Arctan2	200.000	200.002	200.000	200.000	0.000
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	375.244	557.636	474.394	472.044	40.441
	Bounds	424.043	759.68	558.051	552.531	66.168
	Modulo	676.772	1310.041	927.930	901.213	139.013
f_{22}	Arctan2	1889.964	3967.645	2940.667	2814.460	511.266
	Bounds	1836.925	4165.468	3232.624	3254.654	533.029
	Modulo	4033.725	6424.017	5295.7	5264.139	534.114
f_{23}	Arctan2	1945.696	3878.718	2941.584	2855.538	524.918
	Bounds	2229.248	4292.345	3274.916	3234.942	470.076
	Modulo	4120.452	7163.279	5476.301	5403.171	579.628

Table 11: DE/rand/1/bin, $D = 30$, gMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1399.987	-1399.962	-1399.978	-1399.979	0.006
	Bounds	-1399.999	-1399.985	-1399.994	-1399.995	0.003
	Modulo	-1399.961	-1399.859	-1399.917	-1399.918	0.026
f_5	Arctan2	-999.857	-999.669	-999.777	-999.787	0.047
	Bounds	-999.993	-999.962	-999.982	-999.983	0.007
	Modulo	-999.838	-999.54	-999.729	-999.740	0.072
f_9	Arctan2	-599.996	-599.864	-599.954	-599.967	0.040
	Bounds	-600.000	-600.000	-600.000	-600.000	0.000
	Modulo	-599.995	-599.865	-599.956	-599.964	0.025
f_{13}	Arctan2	-200.000	-199.999	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.017	100.002	100.001	0.003
	Bounds	100.000	100.000	100.000	100.000	0.000
	Modulo	100.000	100.007	100.001	100.001	0.001
f_{16}	Arctan2	200.000	200.003	200.001	200.001	0.001
	Bounds	200.000	200.000	200.000	200.000	0.000
	Modulo	200.000	200.003	200.001	200.001	0.001
f_{17}	Arctan2	492.595	539.348	516.238	516.861	9.171
	Bounds	493.27	554.728	528.649	528.595	12.284
	Modulo	495.018	553.231	534.874	534.031	10.433
f_{22}	Arctan2	6441.098	7749.512	7251.214	7288.942	306.046
	Bounds	7170.746	8860.316	8338.675	8387.549	310.100
	Modulo	7005.542	8348.339	7890.777	7895.641	250.596
f_{23}	Arctan2	6414.277	7789.372	7376.831	7406.668	259.184
	Bounds	7193.484	8819.801	8373.7	8387.953	307.687
	Modulo	7230.091	8473.481	7924.702	7926.095	238.693

Table 12: DE/brand/1/bin, $D = 30$, tMT as the PRNG.

Fun.	Type of norm.	Min.	Max	Mean	Med.	Std. dev.
f_1	Arctan2	-1399.972	-1399.886	-1399.939	-1399.943	0.020
	Bounds	-1399.989	-1399.921	-1399.964	-1399.968	0.015
	Modulo	-1399.999	-1399.993	-1399.997	-1399.998	0.001
f_5	Arctan2	-999.896	-999.669	-999.821	-999.837	0.055
	Bounds	-999.942	-999.800	-999.889	-999.894	0.030
	Modulo	-999.996	-999.972	-999.99	-999.991	0.005
f_9	Arctan2	-599.992	-599.864	-599.944	-599.950	0.032
	Bounds	-600.000	-599.937	-599.976	-599.982	0.014
	Modulo	-600.000	-600.000	-600.000	-600.000	0.000
f_{13}	Arctan2	-200.000	-200.000	-200.000	-200.000	0.000
	Bounds	-200.000	-200.000	-200.000	-200.000	0.000
	Modulo	-200.000	-200.000	-200.000	-200.000	0.000
f_{15}	Arctan2	100.000	100.003	100.001	100.000	0.001
	Bounds	100.000	100.002	100.000	100.000	0.001
	Modulo	100.000	100.000	100.000	100.000	0.000
f_{16}	Arctan2	200.000	200.004	200.001	200.001	0.001
	Bounds	200.000	200.003	200.001	200.000	0.001
	Modulo	200.000	200.000	200.000	200.000	0.000
f_{17}	Arctan2	489.020	554.125	532.904	534.463	14.997
	Bounds	508.204	556.453	533.633	534.424	11.959
	Modulo	509.953	553.175	529.219	526.658	10.403
f_{22}	Arctan2	7174.275	8508.328	8005.072	7994.914	298.637
	Bounds	7297.807	8395.703	7887.643	7841.618	262.013
	Modulo	8064.983	9073.263	8629.574	8610.268	236.04
f_{23}	Arctan2	6609.158	8661.705	8051.059	8081.034	351.993
	Bounds	7591.526	8574.803	8082.367	8090.141	212.425
	Modulo	8068.656	9213.194	8734.231	8701.740	275.69